Government College of Engineering, Aurangabad

(An autonomous Institute of Government of Maharashtra)

S.E. (ALL) Examination

End semester Examination

GE241: Engineering Mathematics-III

Time: Three hours 11 NOV 2016 *Max. Marks: 60*

"Verify the Course code and check whether you have got the correct question paper".

N.B.

1. ***All questions are compulsory.***
2. ***Figures to the right indicate full marks.***
3. ***Assume suitable data if necessary and state it clearly.***

Use of non programmable calculator is allowed.

Q.1

(A) Solve any two **(6)**

(i) $\frac{d^2y}{dx^2}+4y = sin^2 2x$ with conditions $y(0) = 0, y'(0) = 0$

(ii) $(D^2 - 2D + 2)y = e^x tanx$, using method of variation of parameter

(iii) $x^2\frac{d^3y}{dx^3}+3x\frac{d^2y}{dx^2}+\frac{dy}{dx} = x^2 logx$

(B) Solve the simultaneous Linear differential equations **(6)**

$$\frac{dx}{dt}+x-2y = 0, \quad \frac{dy}{dt}+x+4y = 0; \qquad given\ that\ x(0) = y(0) = 1$$

Q.2

(A) Solve any two **(6)**

(i) *Form the partial differential by elliminating the arbitrary function f from the equation* $z = x + y + f(xy)$

(ii) *Solve the Lagrange's partial differential equation* $(z-y)p - (x-z)q = x - y$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

(iii) *Solve* $\sqrt{p}+\sqrt{q} = x + y$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

(B) *Solve* $\frac{\partial z}{\partial x} = 3\frac{\partial z}{\partial y}$, *given that* $z(0, y) = 3e^{-y}$ **(6)**

Q.3

Attempt any two **(6)**

(i) *A long column of length l is fixed at one end and is completletely at the free other. If the load p is axially applied to the free end, show that the deflecion curve is*

$$y = \left[1 - cos\left[\left(\sqrt{\frac{p}{EI}}\right)x\right]\right]$$ *where a is the lateral displacement of the free end.*

The deflection of the beam is given by $EI\frac{d^2y}{dx^2} = p[a - y]$

(ii) *An uncharged condenser of capacity C is charged by applying an e.m.f.* $E\sin\left(\frac{t}{\sqrt{LC}}\right)$ *through lead of self inductance L and negligible resistance . Prove that at time t the charge on one of the plates is* $\frac{EC}{2}\left[\sin\left(\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right)\right]$

(iii) *A rectangular plate is bounded by the lines* $x = 0, x = a, y = 0, y = b$ *and the edge temperatures are* $u(0, y) = 0\,, u(x, b) = 0, u(a, y) = 0, u(x, 0) = 5\sin\frac{5\pi x}{a} + 3\cos\frac{3\pi x}{a}$.

Find the steady state temperature distribution at any point of the plate .

Q.4

(A) Attempt any two **(6)**

(i) *Determine the laplace transform of the following functions*

(a) $3e^{-4t} - 5e^{4t}$ (b) $\sin 4t + \cos 4t$ (c) $t^3 + 2t^2 - t + 4$

(ii) *Determine the function* $f(t)$ *whose transform* $F(s)$ *is*

$F(s) = \frac{1}{s}\{2 - 5e^{-s} + 8e^{-3s}\}$. *Sketch the gaph of the function between* $t = 0$ *and* $t = 4$

(iii) *Find the Laplace transform of the periodic function*

$f(t) = e^t, \quad 0 < t < 2\pi \quad and \;\; f(t + 2\pi) = f(t)$

(B) *Solve the equation* $\ddot{x} - 7\dot{x} + 12x = 2$ *using Laplace transform,*

given that at $t = 0\,, x = 1$ *and* $\dot{x} = 5$ **(6)**

Q.5

(A) Attempt any two **(6)**

(i) *Dtermine the tangential and normal components of velocity and acceleration at time* $t = 0$ *of a particle moving along a curve given by* $x = e^{-t}\,, y = 2\cos 3t, z = 2\sin 3t;$ *t is the time .*

(ii) *Find the constants a , b and c so that the the directional derivatives of*

$f = axy^2 + byz + cz^2x^3$ *at* $(1, 2, -1)$ *has a maximum magnitude 64 in a direction parallel to* $y - axis$

(iii) *Prove that* $\bar{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ *isirrotational. Find the scalar function* $f(x, y, z)$ *such that* $\bar{A} = \nabla f$

(B) *Evaluate* $\iint_S \bar{F}\cdot\bar{n}dS$ *over the upper side of the traingle ABC with vertices at the points* $A(1,0,0), B(0,1,0), C(0,0,1)$, *where* $\bar{F} = (x - 2z)\hat{\imath} + (x + 3y + z)\hat{\jmath} + (5x + y)\hat{k}$ **(6)**