Government College of Engineering, Aurangabad

(An autonomous Institute of Government of Maharashtra)

F.E. (Old) Examination

End Semester Examination

GE 101 : ENGINEERING MATHEMATICS - I

Time: Three hours] [Max Marks : 70

"Verify the Course code and check whether you have got the correct question paper"

N.B.

1. **All questions are compulsory**
2. **Figures to the right indicate full marks.**
3. **Assume suitable data if necessary and state it clearly.**
4. **Use of non programmable calculator is allowed**

Q.1. (a) Attempt **any two** of the following **(10)**

(i) Find continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$

(ii) Expand $\cos^6\theta$ in terms of cosine multiples of θ using De-movire's theorem

(iii) If $u+iv=\operatorname{cosec}\left(ix+\frac{\pi}{4}\right)$, prove that $(u^2+v^2)^2=2(u^2-v^2)$

Q.1. (b) Attempt **any one** of the following **(04)**

(i) If $\sin(\theta+i\phi)=\tan\alpha+i\sec\alpha$, show that $\cos 2\theta\cos 2\phi=3$.

(ii) Separate into real and imaginary parts the complex number $\sin^{-1}\left(e^{i\theta}\right)$

Q.2. (a) Attempt **any two** of the following **(10)**

(i) Evaluate $\lim_{x\to 0}\log_x \sin x$

(ii) If $y=e^{\sin^{-1}x}$, then prove that $(1-x^2)y_{n+2}=(2n+1)xy_{n+1}+(n^2+1)y_n$

(iii) Expand $f(x)=2x^3+7x^2+x-1$ in powers of $(x-2)$

Q.2. (b) Attempt **any one** of the following **(04)**

(i) Find n^{th} derivative of $y=\frac{x}{(x-1)(x-2)(x-3)}$

(ii) Evaluate $\lim_{x\to 0}\log_x \sin x$

Q.3. (a) Attempt **any two** of the following **(10)**

(i) If $x=\frac{r}{2}\left(e^{\theta}+e^{-\theta}\right)$, $y=\frac{r}{2}\left(e^{\theta}-e^{-\theta}\right)$, then prove that $\left(\frac{\partial x}{\partial r}\right)_{\theta}=\left(\frac{\partial r}{\partial x}\right)_{y}$.

(ii) If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$

(iii) Find the points on the surface $z^2=xy+1$ nearest to the origin. Also find the distance.

Q.3. (b) Attempt **any one** of the following **(04)**

(i) Determine whether the functions u and v are functionally dependent or not where $u=e^x\sin y$, $v=e^x\cos y$.
If dependent, find the relation between them.

(ii) State Euler's theorem. If $u = \frac{x^3 + y^3}{y\sqrt{x}}$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at $x = 1, y = 2$.

Q.4. (a) Attempt **any two** of the following **(10)**

(i) Solve the system of equations given below

$2x - y - z = 2, \quad x + 2y + z = 2, \quad 4x - 7y - 5z = 2$

(ii) Find rank of the matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

(iii) Find Eigen values and Eigen vectors $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

Q.4. (b) Attempt **any one** of the following **(04)**

(i) Examine whether the vectors given below are linearly independent or dependent: [1,-1,1], [2,1,1], [3,0,2].

(ii) Apply Cayley-Hamilton theorem to A, where $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, and express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

Q.5. (a) Attempt **any two** of the following **(10)**

(i) Find the equation of the cylinder of radius 2 whose axis passing through (1,2,3), and has direction cosines proportional to 2,-3,6.

(ii) Find the equation of the right circular cone whose vertex is at point (0,0,0) and whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and has semivertical angle of 45^0

(iii) Find the equation of the sphere through the circle $x - 2y + 4z = 9$, $x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$ and through the centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$.

Q.5. (b) Attempt **any one** of the following **(04)**

(i) Find the equation of the sphere on the joint of (2,-3,1) and (1,-2,-1)

(ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$