

**Government College of Engineering, Aurangabad**  
(An autonomous Institute of Government of Maharashtra)  
F.E.(All)(Rev) Examination  
End semester Examination  
GE 141: Engineering Mathematics - I

Time: Three hours

28 NOV 2016

Max. Marks: 60

"Verify the course code and check whether you have got the correct question paper".

N.B:-

1. All questions are compulsory
2. Figures to the right indicate full marks
3. Assume suitable data if necessary and state it clearly
4. Use of non programmable calculator is allowed

Q 1: Attempt any three of the following (12)

- a) State DeMoivre's theorem and use it to expand  $\cos 7\theta$  in powers of  $\sin\theta$  &  $\cos\theta$ .
- b) Solve the equation  $(x + i)^8 + x^8 = 0$
- c) If  $\log(\log(x + iy)) = p + iq$  then show that  $y = x \tan(\tan^{-1} p \log \sqrt{x^2 + y^2})$
- d) If  $6 \sinh x + 2 \cosh x + 7 = 0$ , find  $\tanh x$ .

Q 2: Attempt any three of the following (12)

- a) Examine whether the vectors  $X_1, X_2, X_3$  are linearly dependent or independent. If dependent find the relation between them  $X_1 = [3, 1, 1]$ ,  $X_2 = [2, 0, -1]$ ,  $X_3 = [4, 2, 1]$
- b) Test for consistency and solve them if consistent  
 $5x + 3y + 7z = 4$ ,  $2y + 11z + 7x = 5$ ,  $3x + 26y + 2z = 9$
- c) State Cayley Hamilton theorem and use it to find  $A^8$  where  $A = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$
- d) Define the rank of the matrix. Find the rank of the matrix by reducing it to its normal

form  $\begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

Q 3: Attempt any three of the following (12)

- a) State Maclaurin's series, Prove that  $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
- b) If  $y = \cos^4 x$ , find  $\frac{d^{10}y}{dx^{10}}$  at  $x = 0$
- c) Test the convergence of the infinite series  $\sum \frac{n!}{n^n}$
- d) Evaluate  $\lim_{x \rightarrow 0} \left[ \frac{a}{x} - \cot\left(\frac{x}{a}\right) \right]$

Q 4: Attempt any three of the following (12)

- a) find  $\frac{dy}{dx}$  if  $x^y = y^x$  using partial derivatives
- b) if  $z = (x^2 - y^2)f(xy)$  then show that  $\frac{\partial^2 z}{\partial x \partial y} = (x^2 - y^2)\{3f'(xy) + xyf''(xy)\}$
- c) if  $u(x^2y^3, z - 3x) = 0$ , show that  $3x \frac{\partial u}{\partial x} - 2y \frac{\partial u}{\partial y} = 9x$
- d) if  $u = x^2 \sin^{-1}\left(\frac{y}{x}\right) - y^2 \cos^{-1}\left(\frac{x}{y}\right)$ , determine the value of  $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$  at the point (1,1).