Government College of Engineering, Aurangabad

(An autonomous Institute of Government of Maharashtra)

F.E. (C.B.C.S.) (ALL) Examination

End semester Examination November December2016

MA 1001: ENGINEERING MATHEMATICS I

Time: Three hours 28 NOV 2016 *Max. Marks: 60*

"Verify the Course code and check whether you have got the correct question paper".

N.B:-

1. ***All questions are compulsory***
2. ***Figures to the right indicate full marks***
3. ***Assume suitable data if necessary and state it clearly***
4. ***Use of non programmable calculator is allowed***

Q.1: (a) Solve any one **(4)**

(i) Solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$

(ii) Expand $\cos^6\theta - \sin^6\theta$ in terms of cosine multiples of θ

(b) Solve any two **(8)**

(i) If $u = \log\left[\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$ prove that $\cosh u = \sec\theta$

(ii) If $\cos\alpha\cosh\beta = \frac{x}{2}$, $\sin\alpha\sinh\beta = \frac{y}{2}$, prove that $\sec(\alpha - i\beta) + \sec(\alpha + i\beta) = \frac{4x}{x^2 + y^2}$

(iii) If $\tan[\log(x + iy)] = a + ib$, where $a^2 + b^2 \neq 1$, prove that $\tan[\log(x^2 + y^2)] = \frac{2a}{1 - (a^2 + b^2)}$

Q.2: (a) Solve any one **(4)**

(i) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$

(ii) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

(b) Solve any two **(8)**

(i) Determine the value of a and b for which the system

$\begin{bmatrix} 3 & -1 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$ has I. Unique solution II. No solution III. Infinitely many solution.

(ii) Show that the vectors $X_1 = (1,2,4)$, $X_2 = (2,-1,3)$, $X_3 = (0,1,2)$ *and* $X_4 = (-3,7,2)$ are linearly dependent and find the relation between them.

(iii) Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{bmatrix} 13 & -3 & 5 \\ 0 & 4 & 0 \\ -15 & 9 & 7 \end{bmatrix}$

Q.3: (a) Attempt any one (4)

(i) If $x=\sin t, y=\sin pt$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$

(ii) Expand $\log\tan\left(\frac{\pi}{4}+x\right)$ in ascending powers of x

(b) Attempt any two (8)

(i) If $y=\sin^{-1}x$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$

(ii) Test the convergence of the series $\frac{1}{1+\sqrt{2}}+\frac{2}{1+2\sqrt{3}}+\frac{3}{1+3\sqrt{4}}+\ldots$

(iii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{x^{n-1}}{n.3^n}$

Q.4: (a) Attempt any one (4)

(i) If $z=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 y}{\partial y\partial x}=\frac{x^2-y^2}{x^2+y^2}$

(ii) If $x=r\cos\theta, y=r\sin\theta$, prove that $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

(b) Attempt any two (8)

(i) If $z=x^n f\left(\frac{y}{x}\right)+y^{-n}f\left(\frac{x}{y}\right)$, prove that $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=n^2z$

(ii) If $z=\tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=-2\sin^3 z\cos z$

(iii) If $x=uv$ and $y=\frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$

Q.5: (a) Attempt any one (4)

(i) If $u^2(x^2+y^2+z^2)=1$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$

(ii) If $u=\sin(x^2+y^2)$ and $a^2x^2+b^2y^2=c^2$, find $\frac{du}{dx}$

(b) Attempt any two (8)

(i) Determine whether the following functions are functionally dependent or not. If functionally dependent, find the relation between them

$u=x+y-z,\quad v=x-y+z,\quad w=x^2+y^2+z^2-2yz.$

(ii) In calculating the total surface area of a cylinder, error of 1% each are made in measuring height and base radius. Find percentage error in calculating the total surface area.

(iii) Examine maxima and minima of the function $f(x,y)=xy(3a-x-y)$ and find its extreme values

*** *** ***